Spatio-temporal development of instabilities in helical vortices

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SPATIO-TEMPORAL DEVELOPMENT OF INSTABILITIES IN HELICAL VORTICES

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In aeronautics, the dynamics of helical vortices are of interest for many applications. For instance, helicopter wakes exhibit a global transition to the Vortex Ring State (VRS) in steep descent regimes which can lead to a loss of control and crash the apparatus. This transition could be associated with an instability phenomenon and we will argue that it is due to the change of nature from convective to absolute of the pairing instability. For wind turbines, instabilities are also known to be responsible for the turbulence in the far wake. Understanding the spatio-temporal development of the different instabilities is therefore also important in this context. In this work, we consider the particular instability that leads to vortex pairing and address our results in both context of wind turbines and helicopter.

Numerical simulations have shown that several instabilities can occur in helical wakes. Both the three-dimensional elliptic instability and the two-dimensional pairing instability were found to play an important role in helicopter as well as wind turbine wakes (Leishman *et al.* [1], Walther *et al.* [2]). However, the theoretical results known about the stability properties of a helical vortex wake were established for only few model cases: a helical filament (Widnall [3]) or in the limit of large wavenumbers (Okulov [4], Okulov and Sørensen [5]). For the pairing instability, which requires to take into account the geometry of the vortex core, a systematic analysis of the absolute/convective nature of the instability was performed numerically in the case of an array of straight vortices (Brancher and Chomaz [6]) but has still to be done for a helical vortex wake.

In order to focus on the development of the pairing instability and filter out all other instabilities, the helical wake is represented by an array of axisymmetric vortex rings. For small helix pitches, this provides a good approximation of the azimuthal vorticity field, as shown in Figure 1. The instabilities developing along the array of vortex rings are characterized in detail. The linearized Navier-Stokes equations for the perturbations are discretized by means of a spectral method, in order to describe the spatio-temporal evolution of the wave packet generated by a localized initial perturbation. The temporal growth rate of the pairing instability is obtained as a function of the perturbation propagation speed for the different parameters ($\frac{a_0}{R}$, $\frac{a_0}{h}$, Re) of the system. A conclusion is eventually made on the different flow configurations that could lead to the Vortex Ring State, which arises when unstable perturbations (with a positive growth rate) propagate upstream faster than the advection speed of vortices. For wind turbine configurations, the distance (behind the rotor plane) at which the pairing instability initiates will be determined as a function of the parameters, which represents a major issue in the design of wind turbine farms.

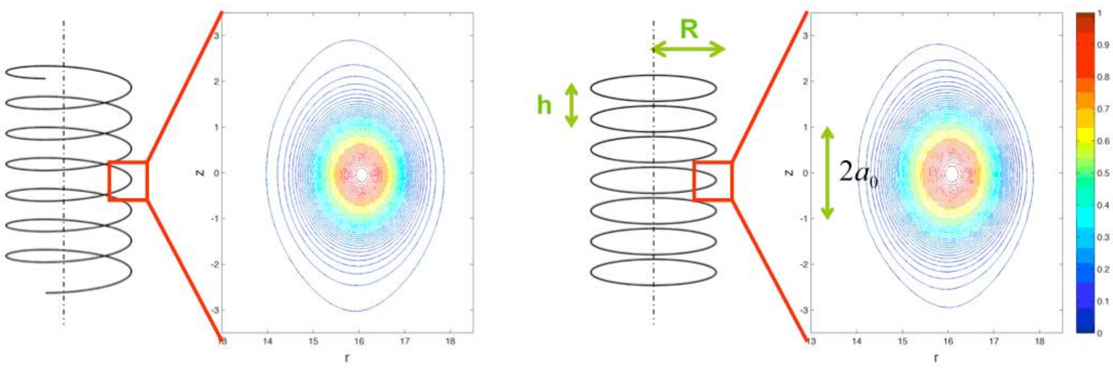
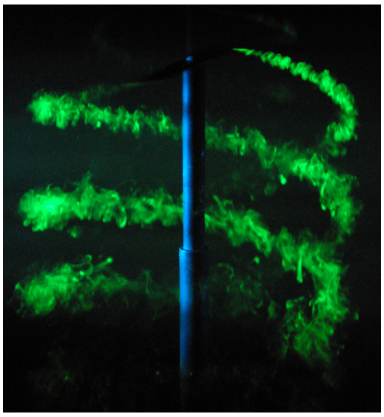


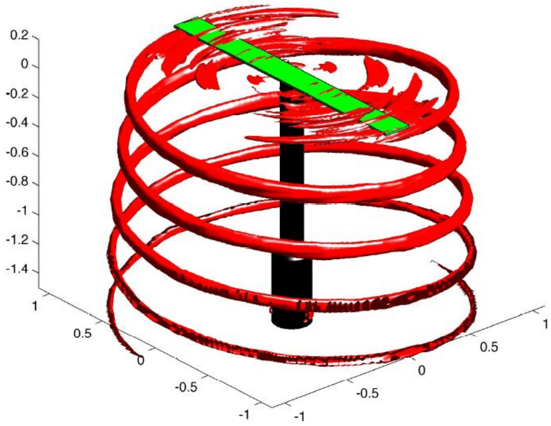
Figure 1: Sketches and azimuthal vorticity contours of a helical vortex (left) and of a vortex rings array (right). The initial condition in both cases is a Gaussian vortex of core radius a_0 and with no axial velocity. Simulations were run for $\frac{a_0}{R} = \frac{1}{16}$ and $\frac{a_0}{h} = \frac{1}{8}$ at $Re = \frac{\Gamma}{2\ \pi\ \nu} = 500$. The helical vortex is computed by forcing the flow to have a helical symmetry (see Delbende [7]).

In parallel with this theoretical approach, experimental results concerning helical wakes will be presented. The characteristics of the wake behind a rotor for different tip speed ratios, defined as the tip speed over the external speed $TSR = \frac{V_{tip}}{V_{ext}}$, are analyzed qualitatively as well as quantitatively.

The facility used here is a closed loop water channel in which an external field can be imposed to the rotor. This free surface channel has a test section of $38 * 50 * 150\ cm^3$ and generates a constant incoming velocity (less than 1 % of turbulence) up to 100 cm/s. A visual access is available on the 5 sides of the test section. The rotor diameter is 16 cm and its rotational speed can be varied up to 10 rps. The sense of rotation determines whether the rotor accelerates the fluid (helicopter climb regime) or if it produces a counter flow (helicopter descent regimes and wind turbines configuration). Qualitative observations of the wake are made through dye visualizations, either by injecting a line of fluorescein upstream of the rotor plane or by painting directly the surface of the blades at the tip location in order to visualize the center of the tip vortices which evolves in the wake downstream as shown in Figure 2(a). A quantitative analysis of the wake is also performed thanks to Stereo Particle Image Velocimetry (SPIV). The vortical structure developing in the wake is identified precisely by calculating the characteristics of the vortices (position, core size, circulation, axial velocity) as a function of their age. For instance, Figure 2(b) shows a three-dimensional reconstruction of the vorticity field obtained by interpolating azimuthally the phase-averaged vorticity calculated in a plane containing the rotor axis.



(a)



(b)

Figure 2: (Left) Dye visualization of the wake illuminated in volume in climb regime at $TSR = 4.5$ The incoming velocity as well as the induced velocity of the rotor are oriented downwards. (Right) Isocontour of vorticity of the 3D wake in climb regime at $TSR = \frac{V_{tip}}{V_{ext}} = 10$ (with $V_{tip} = 100$ cm/s, $V_{ext} = 10$ cm/s and Reynolds number based on the chord and the tip speed $Re = 15000$).

These experimental results are eventually compared with the numerical predictions concerning the threshold of the transition leading to VRS for helicopter configurations and the distance at which the pairing instability occurs in the case of wind turbine wakes.

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